

have work and non-work days specified. Similarly, if a tour does not require consistent daily start time and shift length, daily shifts scheduled (including lengths and start times) in the terminal solution by the values of the decision variables are assigned to work patterns specified by the values of the work pattern variables in the terminal solution.

- 5 Work pattern constraints (e.g. constraint (77)) ensures that a daily shift is scheduled for every agent that will be at work due to the work patterns scheduled in the solution. These daily shifts are assigned to work patterns by following the variable definitions to satisfy tour and shift rules for each tour type. Thus, the weekly tours are stated with off days, work days with daily start times and shift lengths for every agent scheduled in the  
10 solution. Break balance constraints (e.g. constraints (68-70) and (74-76)) ensure that one break of each type is scheduled for every agent scheduled to work on a given day. These breaks are assigned to respective tours that have a given day as a work day while making sure that agent group, tour type, the daily start time, shift length and break window associations specified in the variable definitions are followed correctly to satisfy break  
15 scheduling requirements. Detailed weekly schedules are then assigned to specific agents based on their preferences, starting with the agent having the highest preference priority (e.g. seniority) to complete the rostering process.

- 20 Accordingly, it is to be understood that the embodiments of the invention herein described are merely illustrative of the application of the principles of the invention. It will be clear to one skilled in the art that the above embodiment may be altered or supplemented in many ways without departing from the scope of the invention. Reference herein to details of the illustrated embodiments are not intended to limit the scope of claims, which themselves recite those features regarded as essential to the  
25 invention.

What is claimed is:

- 30 1. A computer implemented method for developing an optimal workforce schedule for a plurality of agent skill groups each with a combination of defined skills to serve a plurality of contact types, part-time/full-time agent work groups with tour and shift scheduling rules located at one or more contact centers each with its own operating hours and time zones, comprising the steps of:
- 35 (a) acquiring net staffing level requirements for each contact type, and for each period to be scheduled by any means;
- (b) acquiring tour, shift, days-off, and break scheduling rules, agent skills groups, part-time/full-time agent work groups, agent availability, and objective criterion to be optimized and its parameters by any means;
- 40 (c) formulating the constraints and objective function of a Mixed Integer Programming (MILP) model to describe the tour and shift rules, staffing level requirements for a plurality of contact types and for each period to be scheduled, agent skills, part-time/full-time agent groups, agent availability, and agent costs;
- 45 (d) solving the LP relaxation of the MILP model formulated in the B&C algorithm, and stopping the B&C and RA algorithms if an optimal solution to the MILP model is found;

(e) calling the RA algorithm if a solution to a node violating only some integrality constraints is found by the B&C algorithm;

(f) applying the RA algorithm to the solution found for the current node by the B&C algorithm, and passing it to the B&C algorithm if an integer feasible solution better than the best integer solution known by the RA algorithm;

2. The method of claim 1 further comprising of repeating steps (e) and (f) for every node the B&C algorithm solves and finds a solution violating only some integrality constraints, until a terminal solution to the MILP model is reached.

3. The method of claim 1 further comprising of processing the terminal solution found to assign daily shifts with start times and shift lengths to work patterns, and days-off scheduled to tours, and daily breaks to specific shifts to develop detailed weekly agent schedules;

4. The method of claim 1 further comprising of assigning agents to detailed weekly tours;

5. The method of claim 1, in which a terminal solution to the MILP model formulated is found when the objective function value for an integer feasible solution differs no more than a pre-specified percentage from the best lower bound found in the B&C algorithm for the MILP model.

6. The method of claim 1, in which a terminal solution to the MILP model formulated is found when an integer feasible solution is found and a pre-specified period of time is passed.

7. The method of claim 1, in which a terminal solution to the MILP model formulated is found when an integer feasible solution is found and a pre-specified number of nodes are solved in the B&C algorithm.

9. A plurality of MILP models for optimal workforce scheduling in a contact center environment involving a plurality of agent skill groups who can serve customers using a plurality of contact types with the definitions of decision variables, formulations of objective functions, agent requirements constraints for each period and contact types to be scheduled, contact type assignment for agents from each agent skill group in each period to be scheduled, constraints to ensure tour, daily shift start time, daily shift length, days-off, work pattern, and a plurality of breaks and associated scheduling rules, and constraints to adhere to the number of agents available in a plurality of part-time/full-time agent work groups.

10. The method of claim 9, in which the objective function, agent requirement constraint for each period and each contact type to be scheduled, and contact type assignment for agents from each agent skill group in each period to be scheduled are mathematically formulated as:

$$\begin{aligned} \text{Minimize } \sum_{i \in I_k} \sum_{k \in K_j} C_{ki}^j X_{ki}^j + \sum_{l \in Q_L k} \sum_{k \in Q_K j} C_{kl}^j Q_{kl}^j \\ + \sum_{k \in Q_K j} \sum_{n \in F_k} \sum_h \sum_{i \in Q_L k} c_{kni}^j Q X_{knh}^j \end{aligned}$$

Subject to

$$\sum_{j \in M_r} e^{jr} G_{ht}^{jr} \geq b_{ht}^r, \quad , r \in R, t \in T_h, h = 1, \dots, 7,$$

$$f_{ht}^j(X, Y, Z, U, W, V, QX, Q, QU, QW, QV) + \sum_{r \in N_j} G_{ht}^{jr} = 0,$$

$$, j \in J, t \in T_h, h = 1, \dots, 7,$$

where

$$\begin{aligned} f_{ht}^j(X, Y, Z, U, W, V, QX, Q, QU, QW, QV) = & \sum_{k \in K_j} \sum_{i \in I_k} a_{kiht} X_{ki}^j \\ & - \sum_{k \in K_j} \sum_{i \in I_k} a_{kiht} (Y_{kih}^j + Y_{ki(h-1)}^j) - \sum_{k \in K_j} \sum_{i \in I_k} \sum_{m \neq h, (h-1)} a_{kiht} Z_{kimh}^j \\ & - \sum_{k \in K_j} \sum_{i \in T1kht} U_{kiht}^j - \sum_{k \in K_j} \sum_{i \in T2kht} (W_{kiht}^j + W_{kih(t-1)}^j) \\ & - \sum_{k \in K_j} \sum_{i \in T3kht} V_{kiht}^j \\ & + \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QI_k} a_{kniht} QX_{kni}^j \\ & - \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QT1kniht} QU_{kniht}^j \\ & - \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QT2kniht} (QW_{kniht}^j + QW_{kniht(t-1)}^j) \\ & - \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QT3kniht} QV_{kniht}^j \end{aligned}$$

$$, j \in J, t \in T_h, h = 1, \dots, 7,$$

where  $X_{ki}^j$  and  $QX_{kni}^j$  are the agents from agent skill group  $j$  assigned to tour type  $k$ , respectively, requiring and not requiring consistent daily start times and shift length with a daily start time of  $i$ ,  $Q_{ki}^j$  is the number of agents from group  $j$  assigned to work pattern  $l$  of tour  $k$ ,  $Y_{kih}^j$  is the number of agents from agent skill group  $j$  assigned to tour type  $k$  with a daily start time of  $i$  and starting their minimum required consecutive days-off on day  $h$ ,  $Z_{kimh}^j$  is the number of agents from agent skill group  $j$  assigned to tour type  $k$  with a daily start time of  $i$  with consecutive days-off starting on day  $m$  and starting their additional days-off (not necessarily consecutive) on day  $h$ ,  $r_k$  is the number of additional days-off an agent assigned to tour  $k$  receives in addition to consecutive days-off,  $U_{kiht}^j$ ,  $W_{kiht}^j$ , and  $V_{kiht}^j$  are the numbers of agents from agent skill group  $j$  assigned to tour type  $k$ ,  $k \in K_j$ , with a daily start time of  $i$  and starting their relief or lunch break in period  $t$  on day  $h$ ,  $B1_{kih}^j$ ,  $B2_{kih}^j$ , and  $B3_{kih}^j$  are the break start times (break window) on day  $h$  for agents from agent skill group  $j$  assigned to tour  $k$  requiring consistent daily start times and shift lengths with a daily start time of  $i$ ,  $QU_{kiht}^j$ ,  $QW_{kiht}^j$ , and  $QV_{kiht}^j$  are the numbers of agents from agent skill group  $j$  assigned to tour type  $k$ ,  $k \in QK_j$ , with a daily start time of  $i$  and starting their relief or lunch break in period  $t$  on day  $h$ ,  $QB1_{kih}^j$ ,  $QB2_{kih}^j$ , and  $QB3_{kih}^j$  are the break start times (break window) on day  $h$  for agents from agent skill group  $j$  assigned to tour  $k$  requiring consistent daily start times and shift lengths with a daily start time of  $i$ ,  $G_{ht}^{jr}$  is the number agents from agent skill group  $j$  assigned to serve customers using type  $r$  contact type in period  $t$  on day  $h$ ,  $b_{ht}^r$  is the number of agents

needed to serve customers using contact type  $r$  in period  $t$  on day  $h$ ,  $M_r$  is the agent skill groups that can serve customers using contact type  $r$ ,  $R$  is the set of all contact types available to customers.

11. The method of claim 9, in which the break and days-off scheduling requirements are formulated into a set of constraints requiring that sufficient number of daily breaks, and pre-specified minimum number of consecutive, and additional non-consecutive days-off are scheduled for agents from each agent skill group assigned to tours requiring consistent daily shift start times as:

$$\begin{aligned}
 10 \quad & X_{ki}^j - Y_{kih}^j - Y_{ki(h-1)}^j - \sum_{m \neq h, (h-1)} Z_{kimh}^j = \sum_{t \in B1kih} U_{kiht}^j, \quad i \in I_k, k \in K_j, h = 1, \dots, 7, \\
 & X_{ki}^j - Y_{kih}^j - Y_{ki(h-1)}^j - \sum_{m \neq h, (h-1)} Z_{kimh}^j = \sum_{t \in B2kih} W_{kiht}^j, \quad i \in I_k, k \in K_j, h = 1, \dots, 7, \\
 & X_{ki}^j - Y_{kih}^j - Y_{ki(h-1)}^j - \sum_{m \neq h, (h-1)} Z_{kimh}^j = \sum_{t \in B3kih} V_{kiht}^j, \quad i \in I_k, k \in K_j, h = 1, \dots, 7, \\
 & r_k Y_{kih}^j = \sum_{l \neq h, (h+1)} Z_{kihl}^j, \quad i \in I_k, k \in K_j, h = 1, \dots, 7, \\
 & Z_{kihm}^j \leq Y_{kih}^j, \quad i \in I_k, k \in K_j, h = 1, \dots, 7, \\
 15 \quad & m \neq h, (h+1) \\
 & X_{ki}^j = \sum_h Y_{kih}^j, \quad i \in I_k, k \in K_j.
 \end{aligned}$$

12. The method of claim 9, in which the break and daily shift scheduling requirements are formulated into a set of constraints requiring that sufficient number of daily breaks and shifts are scheduled for agents from each agent skill group assigned to tours not requiring consecutive days-off and consistent daily shift start times as:

$$\begin{aligned}
 25 \quad & QX_{knh}^j = \sum_{t \in QB1knh} QU_{knh}^j, \quad n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \\
 & QX_{knh}^j = \sum_{t \in QB2knh} QW_{knh}^j, \quad n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \\
 & QX_{knh}^j = \sum_{t \in QB3knh} QV_{knh}^j, \quad n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \\
 & \sum_{l \in QLK} A_{klh} Q_{kl}^j = \sum_{n \in F_k} \sum_{i \in I_k} QX_{knh}^j, \quad k \in QK_j, h = 1, \dots, 7.
 \end{aligned}$$

13. The method of claim 9, in which daily breaks, shift and days-off scheduling requirements for tours requiring consistent daily shift start times are formulated into a set of constraints by defining pseudo tours  $k$  in  $QK_j$ , each with one daily start time in  $QI_k$  and shift lengths in  $F_k$  as:

$$\begin{aligned}
 35 \quad & QX_{knh}^j = \sum_{t \in QB1knh} QU_{knh}^j, \quad n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \\
 & QX_{knh}^j = \sum_{t \in QB2knh} QW_{knh}^j, \quad n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \\
 & QX_{knh}^j = \sum_{t \in QB3knh} QV_{knh}^j, \quad n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \\
 & \sum_{l \in QLK} A_{klh} Q_{kl}^j = \sum_{n \in F_k} \sum_{i \in I_k} QX_{knh}^j, \quad k \in QK_j, h = 1, \dots, 7.
 \end{aligned}$$

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14. The method of claim 9, in which the limited agent availability for agent group  $j$  and tour  $k$ ,  $D_k^{j\max}$  and  $QD_k^{j\max}$ , is formulated into a set of constraints as:

$$\begin{aligned} \sum_{i \in I_k} X_{ki}^j &\leq D_k^{j\max}, & j \in J, k \in K_j, \\ \sum_{l \in Q_k} Q_{kl}^j &\leq QD_k^{j\max}, & j \in J, k \in QK_j. \end{aligned}$$

15. The method of claim 9, in which fixed days off or early closure days for a tour is assured by setting values of associated days off variables to zero.

16. The method of claim 9, in which the objective function can be formulated to minimize cost, total agent time scheduled, total agent time scheduled weighted by agent skill group, total paid agent time scheduled, total paid agent time scheduled weighted by agent skill group, or to maximize agent preferences.

17. The method of claim 9, in which the objective function, and agent requirement constraint for each period and contact types are formulated to include an overage and an underage variable, and a penalty for each period and contact type with agent shortage in the objective function when there aren't enough number of agents to meet requirements by all contact types in all periods to be scheduled.

18. The method of claim 17, in which the total overage or underage for a plurality of contact types is limited to a pre-specified level.

19. A rounding algorithm (RA) to search for integer feasible solutions using a solution violating only some integrality requirements on some decision variables to a Mixed Integer Linear Programming (MILP) model of a workforce scheduling environment comprising the step of:

- a) Obtaining the values of the decision variables in the solution found to the MILP model;
- b) Rounding the fractional values of decision variables  $U$ ,  $W$ ,  $V$ ,  $QX$ ,  $QU$ ,  $QW$ ,  $QV$ , AND  $G$  down, and weekly tour variables  $X$ , and work pattern variables down if their fractional part is less than or equal to 0.50, and up if greater than 0.50, provided the agent availability constraints are satisfied;
- c) Scheduling additional days off by increasing the values of  $Y$  and  $Z$  if additional days off needed to satisfy tour scheduling requirements, and additional daily shifts to have a shift scheduled for every agent who will be working on a given day based of the work pattern scheduled;
- d) Scheduling additional daily breaks if the number of breaks of each type is not sufficient to satisfy break scheduling rules for a tour, and unscheduling breaks if more breaks are scheduled due to rounding;
- e) Computing agent shortages and excesses for each contact type and planning period;
- f) Computing agent availability for all agent groups, comparing them with the agent allocations to different contact types by the rounded values of allocation variables  $G$ , and allocating excess agents not accounted by allocation variables  $G$  to contact types;
- g) Checking the solution constructed in steps (a) through (f) to find out if all agent requirements are met in every period and , when all requirements are

met, eliminating all redundant agent tours and stopping with the integer feasible solution found;

- h) Stopping with an infeasible solution if all available agents are scheduled;
- i) Continuing to step (i) if there are agents available to schedule and there shortages in some periods for some contact types;
- j) Finding all periods in which some contact types have shortages, finding an agent together with a complete tour schedule with work and non-work days, daily shift start times and shift lengths, daily break times to reduce the agent shortages, and adding to the solution by updating the values of the decision variables to include the newly found tour schedule;
- k) Repeating steps (h), (i), and (j) until an integer feasible solution satisfying all requirements if found;
- l) Examining the tours scheduled in the integer feasible solution found and eliminating redundant tours

**20.** A computer program in a computer readable medium to be run on a computer for developing optimal schedules for a plurality of agents with a combination of defined skills, and work groups with part-time/full-time work requirements, a plurality of contact types requiring defined agent skills, a plurality of tour and shift scheduling rules, and a plurality of contact centers, comprising of:

- means for acquiring agent and skill requirements for each period to be scheduled;
- means for acquiring, for each contact center to be scheduled, operating hours, agent skill groups, agent work groups specifying part-time/full-time work patterns, and scheduling paradigm for contact centers to specify whether each center is to be scheduled independently or a plurality of contact centers as a virtual contact center with combined;
- means for acquiring tour and shift parameters and scheduling rules, and their availability to agent skill and work groups at different contact centers;
- means for acquiring the objective criterion to be optimized and objective function parameters to quantify cost or benefit;
- means for generating an MILP model by defining decision variables, parameters and sets, and then formulating the objective function, agent and skill requirement constraints, breaks constraints, days-off and work pattern constraints, agent availability constraints, other schedule related constraints, and non-negativity and integrality constraints;
- means for solving the MILP model using an optimization algorithm;
- means for generating a detailed tour schedule using a terminal solution to the MILP model found by assigning daily shifts to work patterns scheduled, days-off to weekly tours, and breaks to daily shifts;
- means for assigning agents to specific tours.